## Fragments of the Books on Arithmetic. 1194

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What is mathematics?

Aristotle thinks that all philosophy consisted of theory and practice, <sup>1195</sup> and divides the practical into ethical and political, and the theoretic again into the theological, the physical, and the mathematical. And thus very clearly and skilfully he shows that mathematics is (a branch of) philosophy.

The Chaldæans were the originators of astronomy, and the Egyptians of geometry and arithmetic....

And whence did mathematics derive its name?

Those of the Peripatetic school affirmed that in rhetoric and poetry, and in the popular music, any one may be an adept though he has gone through no process of study; but that in those pursuits properly called studies, 1196 none can have any real knowledge unless he has first become a student of them. Hence they supposed that the theory of these things was called *Mathematics*, from μάθημα, study, science. And the followers of Pythagoras are said to have given this more distinctive name of mathematics to geometry, and arithmetic alone. For of old these had each its own separate name; and they had up till then no name common to both. And he (Archytas) gave them this name, because he found science 1197 in them, and that in a manner suitable to man's study. 1198 For they (the Pythagoreans) perceived that these studies dealt with things eternal and immutable and perfect, 1199 in which things alone they considered that science consisted. But the more recent philosophers have given a more extensive application to this name, so that, in their opinion, the mathematician deals not only with substances 1200 incorporeal, and falling simply within the province of the understanding, <sup>1201</sup> but also with that which touches upon corporeal and sensible matter. For he ought to be cognisant of 1202 the course of the stars, and their velocity, and their magnitudes, and forms, and distances. And, besides, he ought to investigate their dispositions to vision, examining into the causes, why they are not seen as of the same form and of the same size

<sup>1194</sup> Fabricius, Biblioth. Græca, ed. Harles, vol. iii. p. 462. Hamburg, 1793.

<sup>1195</sup> θεωρίας καὶ πράξεως.

<sup>1196</sup> μαθήματα.

<sup>1197</sup> τὸ ἐπιστημονικόν.

<sup>1198</sup> μάθησιν.

<sup>1199</sup> εἰλικρινῆ, absolute.

<sup>1200</sup> ὕλην.

<sup>1201</sup> νοητήν.

<sup>1202</sup> θεωρητικός.

from every distance, retaining, indeed, as we know them to do, their dispositions relative to each other, <sup>1203</sup> but producing, at the same time, deceptive appearances, both in respect of order and position. And these are so, either as determined by the state of the heavens and the air, or as seen in reflecting and all polished surfaces and in transparent bodies, and in all similar kinds. In addition to this, they thought that the man ought to be versed in mechanics and geometry and dialectics. And still further, that he should engage himself with the causes of the harmonious combination of sounds, and with the composition of music; which things are bodies, <sup>1204</sup> or at least are to be ultimately referred to sensible matter.

What is mathematics?

Mathematics is a theoretic science <sup>1205</sup> of things apprehensible by perception and sensation for communication to others. <sup>1206</sup> And before this a certain person indulging in a joke, while hitting his mark, said that mathematics is that science to which Homer's description of Discord may be applied.—

"Small at her birth, but rising every hour,

While scarce the skies her horrid (mighty) head can bound,

She stalks on earth and shakes the world around." 1207

For it begins with a point and a line,  $^{1208}$  and forthwith it takes heaven itself and all things within its compass.

How many divisions are there of mathematics?

Of the more notable and the earliest mathematics there are two principal divisions, viz., arithmetic and geometry. And of the mathematics which deals with things sensible there are six divisions, viz., computation (practical arithmetic), geodesy, optics, theoretical music, mechanics, and astronomy. But that neither the so-called tactics nor architecture, <sup>1209</sup> nor the popular music, nor physics, nor the art which is called equivocally the mechanical, constitutes, as some think, a branch of mathematics, we shall prove, as the discourse proceeds, clearly and systematically.

As to the circle having eight solids and six superficies and four angles....What branches of arithmetic have closest affinity with each other? Computation and theoretical music have

<sup>1203</sup> τοὺς πρὸς ἄλληλα λόγους.

<sup>1204</sup> σώματα, substances.

<sup>1205</sup> ἐπιστήμη θεωρητική.

<sup>1206</sup> πρὸς τὴν τῶν ὑποπιπτόντων δόσιν.

<sup>1207</sup> Iliad, iv. 442-443 (Pope).

<sup>1208</sup> σημείου καὶ γραμμῆς.

<sup>1209</sup> τὸ ἀρχιτεκτονικόν.

a closer affinity than others with arithmetic; for this department, being one also of quantity and ratio, approaches it in number and proportion. Optics and geodesy, again, are more in affinity with geometry. And mechanics and astrology are in general affinity with both.

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As to mathematics having its principles <sup>1211</sup> in hypothesis and about hypothesis. Now, the term hypothesis is used in three ways, or indeed in many ways. For according to one usage of the term we have the dramatic revolution; <sup>1212</sup> and in this sense there are said to be hypotheses in the dramas of Euripides. According to a second meaning, we have the investigation of matters in the special in rhetoric; and in this sense the Sophists say that a hypothesis must be proposed. And, according to a third signification, the beginning of a proof is called a hypothesis, as being the begging of certain matters with a view to the establishment of another in question. Thus it is said that Democritus <sup>1213</sup> used a hypothesis, namely, that of atoms and a vacuum; and Asclepiades <sup>1214</sup> that of atoms <sup>1215</sup> and pores. Now, when applied to mathematics, the term hypothesis is to be taken in the third sense.

That Pythagoras was not the only one who duly honoured arithmetic, but that his best known disciples did so too, being wont to say that "all things fit number." <sup>1216</sup>

That arithmetic has as its immediate end chiefly the theory of science, <sup>1217</sup> than which there is no end either greater or nobler. And its second end is to bring together in one all that is found in determinate substance. <sup>1218</sup>

Who among the mathematicians has made any discovery?

<sup>1210</sup> ἀναλογίας.

<sup>1211</sup> ἀρχάς, beginnings.

<sup>1212</sup> περιπέτεια, reversal of circumstances on which the plot of a tragedy hinges.

<sup>1213</sup> A native of Abdera, in Thrace, born about 460 b.c., and, along with Leucippus, the founder of the philosophical theory of atoms, according to which the creation of all things was explained as being due to the fortuitous combination of an infinite number of atoms floating in infinite space.

<sup>1214</sup> A famous physician, a native of Bithynia, but long resident in great repute at Rome in the middle of the first century b.c. He adopted the Epicurean doctrine of atoms and pores, and tried to form a new theory of disease, on the principle that it might be in all cases reduced to obstruction of the pores and irregular distribution of the atoms.

<sup>1215</sup> ὄγκοις.

<sup>1216 [</sup>Wisd. xi. 20; Ecclus. xxxviii. 29 and xlii. 7.]

<sup>1217</sup> τὴν ἐπιστημονικὴν θεωρίαν.

<sup>1218</sup> συλλήβδην καταλαβεῖν πόσα τῆ ώρισμένη οὐσία συμβέβηκεν.

Eudemus<sup>1219</sup> relates in his *Astrologies* that Œnopides<sup>1220</sup> found out the circle of the zodiac and the cycle<sup>1221</sup> of the great year. And Thales<sup>1222</sup> discovered the eclipse of the sun and its period in the tropics in its constant inequality. And Anaximander<sup>1223</sup> discovered that the earth is poised in space, <sup>1224</sup> and moves round the axis of the universe. And Anaximenes<sup>1225</sup> discovered that the moon has her light from the sun, and found out also the way in which she suffers eclipse. And the rest of the mathematicians have also made additions to these discoveries. We may instance the facts—that the fixed stars move round the axis passing through the poles, while the planets remove from each other<sup>1226</sup> round the perpendicular axis of the zodiac; and that the axis of the fixed stars and the planets is the side of a pentedecagon with four-and-twenty parts.

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<sup>1219</sup> A native of Rhodes, a disciple of Aristotle, and editor of his works.

<sup>1220</sup> A native of Chios, mentioned by Plato in connection with Anaxagoras, and therefore supposed by some to have been a contemporary of the latter sage.

<sup>1221</sup> περίστασιν, revolution.

<sup>1222</sup> Of Miletus, one of the sages, and founder of the Ionic school.

<sup>1223</sup> Of Miletus, born 610 b.c., the immediate successor of Thales in the Ionic school of philosophy.

<sup>1224</sup> μετέωρος.

<sup>1225</sup> Of Miletus, the third in the series of Ionic philosophers.

<sup>1226</sup> απεχουσιν άλλήλων.